

Glueball Masses in Relativistic Potential Model

A. Shpenik*, Yu. Fekete†, J. Kis‡

*Uzhgorod State University,
Department of Theoretical Physics, Voloshin str. 32,
88000 Uzhgorod, Ukraine*

Abstract

The problem of glueball mass spectra using the relativistic Dirac equation is studied. Also the Breit-Fermi approach used to obtaining hyperfine splitting in glueballs. Our approach is based on the assumption, that the nature and the forces between two gluons are the short-range. We were to calculate the glueball masses with used screened potential.

**e-mail address:* shpenik@org.iep.uzhgorod.ua

†*e-mail address:* kishs@univ.uzhgorod.ua

1 Introduction

The existence of gluon self-coupling in QCD suggests that, in addition to the conventional qq states, there may be non-qq mesons - bound states including gluons (glueballs and qqg hybrids). The existence of glueball states made from gluons is one of the important predictions of QCD. The discovery of these glueball states would be the strong support to the QCD theory. Therefore the search and identification of glueballs have been a very attractive research task. The abundance of qq mesons and possible mixing of glueballs and ordinary mesons make the current situation with the identification of glueball states rather complicated. So the spectrum of QCD is expected to contain glueballs hybrids and multiquark states. However, the theoretical quittance on the properties of unusual states is often contradictory, and models that agree in qq sector differ in their predictions about new states. Moreover, the abundance of qq meson states in the region $1 - 2\text{GeV}$ and glueball-quarkonium mixing makes the identification of the lightest non-qq mesons difficult. Whereas there is a general agreement that the lightest glueball should have quantum numbers 0^{++} its mass remains controversial. The one of the possibilities to distinguish between possibilities is to study of production rates and decays branching ratios which are expected to be sensitive to the constituent structure. Glueballs are preferentially expected in production of gluon rich environment such as J/Ψ -, central production in hadron - hadron collisions by double Pomeron exchange and pp annihilation into mesons near threshold, or jj collision.

The quark model predicts that there are two mesons with $I = 0$ in the 3P_0 qq nonet but apart from the states $f_j(1710)$ with $J = 0$ or (and) 2, four states $f_0(400 - 1200)$, $f_0(980)$, $f(1370)$ and $f_0(1500)$ are listed by Particle Data Group. There are too many controversies about these states. One of the possibilities is to put aside the $f_0(400 - 1200)$ and $f_0(980)$ and focus on $f(1370)$, $f_0(1500)$, $f_j(1710)$ although the spin parity of the $f_j(1710)$ is $J^{PC} = 0^{++}$ or (and) 2^{++} controversial. Recently several authors study the quarkonia-glueball contents of the $f(1370)$, $f(1500)$, $f_j(1710)$ by studying the mixing of these three states. The same situation is also in the case of identification of the 2^{++} glueball. Some progress has been made recently in the 0^{++} scalar and 2^{++} tensor glueball sector, where both experimental and QCD lattice simulation results seem to converge.

In this paper we will study the problem of glueball mass spectra using the relativistic Dirac equation, which was used to the calculations of mesons spectra, and the Breit-Fermi approach which was also used to obtaining the hyperfine splitting in different mesons. Our approach is based on the assumption that the nature and the forces between two gluons are the same that between two quarks, and because that the chosen approaches in both cases must be similar. In our previous papers we obtained a good description of meson spectra, fine and hyperfine splitting in relativistic Dirac approach with screened potential. Because that we were able to calculate the glueball masses and decays in the same approach, with the same parameters.

2 Method Calculation of Glueballs states

For calculation of glueball mass-spectrum we used Dirac equation with mixing Lorentz structure of potential for glueballs with spin averaged masses

$$\begin{aligned} -\frac{dF_{ij}(r)}{dr} + \frac{k}{r}F_{ij}(r) &= [E - m - V]G_{ij}(r) \\ \frac{dG_{ij}(r)}{dr} + \frac{k}{r}G_{ij}(r) &= [E + m - V]F_{ij}(r) \end{aligned} \quad (1)$$

where

$$\vec{\sigma} \cdot \vec{\ell} = \begin{pmatrix} j - \frac{1}{2} \\ -j - \frac{3}{2} \end{pmatrix} = -(1 + k). \quad (2)$$

Let us write to equation all possible potential type

$$\begin{aligned} i \frac{\partial \Psi(r, t)}{\partial t} &= \\ &= \left(\frac{\alpha}{i} \nabla + \beta \left[m + V^S(r) + \gamma^5 V^{PS}(r) + \gamma^\mu V_\mu^V(r) + \gamma^\mu \gamma^5 V_\mu^A(r) + \sigma^{\mu\eta} V_{\mu\eta}^T \right] \right). \end{aligned} \quad (3)$$

The vector and scalar part we present in the following form:

$$\begin{aligned} V_V(r) &= V_{OGE}(r) + \varepsilon V_{conf.}(r) \\ V_S(r) &= (1 - \varepsilon) V_{conf.}(r) \end{aligned} \quad (4)$$

and substituted it to (1)

$$\begin{aligned} (E - V_{OGE}(r) - V_{conf.}(r) - m)F(r) &= -\frac{k}{r}G(r) - \frac{dG(r)}{dr} \\ (E - V_{OGE}(r) + (1 - 2\varepsilon)V_{conf.}(r) + m)G(r) &= -\frac{k}{r}F(r) + \frac{dF(r)}{dr} \end{aligned} \quad (5)$$

$$\begin{aligned} (E - V_{OGE}(r) - V_{conf.}(r) - m)F(r) &= \\ &= -\frac{k}{r} \frac{-\frac{k}{r}F(r) + \frac{dF(r)}{dr}}{(E - V_{OGE}(r) + (1 - 2\varepsilon)V_{conf.}(r) + m)}, \end{aligned} \quad (6)$$

$$\begin{aligned} (E - V_{OGE}(r) - V_{conf.}(r) - m)F(r) &= \\ &= -\frac{k}{r} \frac{-\frac{k}{r}F(r) + \frac{dF(r)}{dr}}{(E - V_{OGE}(r) + (1 - 2\varepsilon)V_{conf.}(r) + m)}. \end{aligned} \quad (7)$$

We obtain for the small component of the wave function $G(r)$:

$$\begin{aligned} \frac{dG(r)}{dr} &= -(E - V_{OGE}(r) - V_{conf.}(r) - m)F(r) - \\ &- \frac{k}{r} \left[\frac{-\frac{k}{r}F(r) + \frac{dF(r)}{dr}}{(E - V_{OGE}(r) + (1 - 2\varepsilon)V_{conf.}(r) + m)} \right], \end{aligned} \quad (8)$$

$$[(1 - 2\varepsilon)V_{conf.}(r) - V_{OGE}(r)]G(r) +$$

$$\begin{aligned}
& + (E + (1 - 2\varepsilon) V_{conf.}(r) - V_{OGE}(r) + m) \frac{dG(r)}{dr} = \\
& = \frac{k}{r^2} F(r) - \frac{k}{r} F'(r) + F''(r).
\end{aligned} \tag{9}$$

Into the formula (5) we substitute $G(r)$, so we obtain the second order equation for the wave function $F(r)$:

$$\begin{aligned}
& F''(r) - \frac{[(1 - 2\varepsilon) V_{conf.}(r) - V_{OGE}(r)]'}{(E + (1 - 2\varepsilon) V_{conf.}(r) - V_{OGE}(r) + m)} F'(r) + \\
& + \left\{ \frac{k}{r^2} - \frac{k^2}{r^2} + \frac{[(1 - 2\varepsilon) V_{conf.}(r) - V_{OGE}(r)]'}{(E + (1 - 2\varepsilon) V_{conf.}(r) - V_{OGE}(r) + m)} \frac{k}{r} + \right.
\end{aligned} \tag{10}$$

$$\left. + (E + (1 - 2\varepsilon) V_{conf.}(r) - V_{OGE}(r) + m) (E - V_{OGE}(r) - V_{conf.}(r) - m) \right\} F(r) = 0.$$

As for the interaction potential we choose the same screened potential as in the case of Breit-Fermi equation. With the same interaction potential we obtained good results in meson spectroscopy, and because that we try to apply this approach to glueballs. In the frame of Breit-Fermi approach the spin - spin interaction term is

$$V_{SS} = \frac{2}{3m_{q_1}m_{q_2}} \vec{S}_1 \vec{S}_2 \Delta V_V. \tag{11}$$

We suggest for calculation to use configuration interaction approach, which was very successfully applied in atomic physics.

3 Results

In the Table 2 we show the results obtained in potential model for spin average glueball masses in comparison with the data of other authors. As it is seen from this table for the lowest glueball state our result is close for to the results of other authors. In our model the first and second orbital as well radial excited states are lower then in the lattice calculation [1], [2]. The same situation is also in the Table 3, where we show the results which incorporated the spin-spin effects. These probably mean, that the strong interaction between two gluons in glueball more strong than between two quarks in mesons. It is interesting to note, that to a same conclusion come J. Cui and H. Jin [3] during calculation of glueball masses, using Bethe-Salpeter equation.

Table 1. Spin averaged glueball masses from Dirac equation with ($V_0 = 0.586 GeV$).

Quantum numbers	Our results $M (GeV)$	Nonperturb.method [4]	Lattice data [1]	Lattice data [2]
$\ell = 0, n_r = 0$	4.68	4.68	4.66	4.55
$\ell = 1, n_r = 0$	5.84	6.0	6.36	6.1
$\ell = 0, n_r = 1$	6.22	6.0	6.68	6.45
$\ell = 2, n_r = 0$	6.69	7.0	9.0	7.7
$\ell = 1, n_r = 1$	6.96	8.0		

Table 2. Glueball masses from Breit-Fermi approach in comparison with other authors.

J^{PC}	<i>our results, M (GeV)</i>	<i>ref.[4]</i>	<i>ref.[2]</i>	<i>ref.[1]</i>	<i>ref.[5]</i>	<i>ref.[6]</i>	<i>ref.[7]</i>
0^{++}	1.70	1.58	1.73	1.74	1.645	1.686	1.659
2^{++}	2.19	2.59	2.40	2.47	2.337	2.380	2.304
3^{++}	3.24	3.58	3.69	4.3			

The Decays of Glueballs to two light mesons:

$$\Gamma(1^3S_1 \rightarrow XX) = 55MeV,$$

$$\Gamma(2^3S_1 \rightarrow XX) = 24MeV,$$

$$\Gamma(3^3S_1 \rightarrow XX) = 15MeV.$$

4 Discussion and conclusion

As it is note by Minkowski and Ochs [8], [11] in the gluon jet in case of triplet neutralization the leading particles are the hadrons formed by the primary gluon and soft qq pairs. If there are hybrid mesons they may be formed in the fast color singlet qgg system alternatively the fast hadrons are ordinary qq mesons formed at the end of the parton cascade after having absorbed all gluon energy. If the octet mechanism is at work the leading gluon may also form a glueball. On the other hand in the quark jet neither the hybrid nor the glueball will be leading if the leading quark is only neutralized in color by a soft antiquark.

Table 3. Production of leading hadrons in the jet according [8].

	<i>neutralization</i>	<i>qq</i>	<i>hybrid</i>	<i>glueball</i>
<i>quark jet</i>	<i>triplet</i>	<i>yes</i>	<i>no</i>	<i>no</i>
<i>gluon jet</i>	<i>triplet</i>	<i>yes</i>	<i>yes</i>	<i>no</i>
	<i>octet</i>	<i>no</i>	<i>no</i>	<i>yes</i>

As it is note by Burakovsky [9] it is widely believed that pseudoscalar mesons are the Goldstone bosons of broken $SU(3) \times SU(3)$ chiral symmetry of QCD, and that they should be massless in the chirally symmetric phase. Because that it is not clear how the resonance spectrum would be suitable for the description of the pseudoscalar mesons.

It is firmly established that the lightest glueball state is the scalar glueball. According to Burakovsky, who obtained the glueball masses from the Regge phenomenology, with linear Regge trajectory with negative intercept

$$M^2(0^{++}) = \frac{3}{\sqrt{2}}M(\rho)$$

and

$$M^2(2^{++}) = \sqrt{2}M(0^{++})$$

So $M(0^{++}) = 1620 MeV$, $M(2^{++}) = 2290 MeV$.

Tensor glueball is the lowest resonance lying on the Pomeron trajectory with unit intercept. As for this glueball state in PDG corresponds three candidates in this mass region $f_j(2220)$, $J = 2$ or 4 , $f_2(2300)$ and $f_2(2340)$.

As to the question of hybrids. If hybrid spectroscopy works like conventional mesons spectroscopy or glueball spectroscopy than as shown by Toussaint [10], we expect that the lowest multiplet of hybrids contains 1^{-+} , 1^{-} , 0^{-+} , 2^{-+} particles both isovector and isoscalar which will be split by color hyperfine interaction. In this paper we restrict ourselves only with the calculations of glueball masses.

Using a non-perturbative method based on asymptotic behavior of Wilson loops A. Kaidalov and Yu. Simonov calculated the masses of glueballs and corresponding Regge trajectories. These results are shown in the Tables 1, 2.

The difficulties in identification of the glueball states, and the poor experimental data makes the investigation of the problem for glueballs actually. We try to obtain the mass spectrum of glueballs and the decay width in potential model approach. Using the same method and the same parameters as in the case of meson spectra calculation we obtained for glueball masses a good agreement with the possible experimental data. We obtained the results for spin average glueball states and the results which take into account the spin-spin interaction. Moreover, our results, as well as other authors results, shown, that the lowest scalar glueball mass is $M(0^{++}) = 1700 \text{ MeV}$, and the tensor glueball amass is $M(2^{++}) = 2290 \text{ MeV}$. Other results in Tables 1, 2 are predictions.

References

- [1] M. Teper, hep-th/9812187
- [2] C.J. Morningstar and M. Peardon, Nucl. Phys. B (Proc. Suppl.) 63 A-C (1998022); Phys. Rev. **D60**(1999) 034509
- [3] J.Y. Cui H.Y. Jin, J.M. Wu, The Decays of Glueballs to two light mesons, hep-ph/9803276
- [4] A.B. Kaidalov, Yu.A. Simonov, Phys.Lett. **B477** (2000) 163-170
- [5] UKQCD Coll. (G. S. Bali et al.), Phys. Lett. **B309** (1993) 378
- [6] H. Chen, Sexton A. Vaccarino and D. Weingarten hep-lat/9308010, Nucl. Phys. B (Proc. Suppl.) **34**, 357 (1993); W. Lee and D. Weingarten, hep-lat 9805029 Nucl. Phys. (Proc. Suppl.) **63** (1998) 194; A. Vaccarino and D. Weingarten in preparation
- [7] C.J. Morningstar and M. Peardon, Phys. Rev. **D56** (1997) 4043
- [8] Peter Minkowski, Wolfgang Ochs, Phys.Lett. **B485** (2000) 139-144
- [9] L. Burakovsky, Glueball Spectroscopy in Regge Phenomenology Found.Phys. **28** (1998) 1595-1605
- [10] Toussaint, Nucl.Phys.Proc.Suppl. 83-84 (2000) 151-155 p.
- [11] Wolfgang Ochs, Lightest glueball and scalar meson nonet in production and decay, hep-ph/9909241